Exercice n° 3 :

1] $z_1 = 1 + i\sqrt{3}$; $z_2 = \sqrt{3} - i$

$|z_1| = 2$; $\begin{cases} \cos\theta = \frac{1}{2} \\ \sin\theta = \frac{\sqrt{3}}{2} \end{cases} \Rightarrow \theta \equiv \frac{\pi}{3} \ [2\pi]$

$|z_2| = \sqrt{\sqrt{3}^2 + (-1)^2} = 2$ $\begin{cases} \cos\theta = \frac{\sqrt{3}}{2} \\ \sin\theta = \frac{-1}{2} \end{cases} \Rightarrow \theta \equiv -\frac{\pi}{6} \ [2\pi]$

Alors $z_1 = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

$z_2 = 2\left(\cos\frac{-\pi}{6} + i\sin\frac{-\pi}{6}\right)$

2] $z_1^{21} = 2^{21}\underbrace{\left(\cos 7\pi + i\sin 7\pi\right)}_{(-1)} = 2^{21} \times (-1)$

$= -2^{21}$

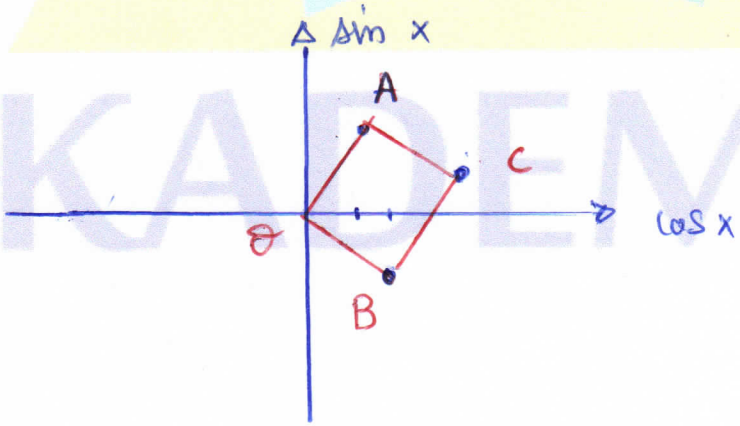
$z_1^{21} = \left(1 + i\sqrt{3}\right)^{21}$

3] a) $A \longmapsto z_1 = 1 + i\sqrt{3}$

$B \longmapsto z_2 = \sqrt{3} - i$

$C \longmapsto Z = z_1 + z_2 = (1+\sqrt{3}) + i(\sqrt{3} - 1)$

b)



$z_{\overrightarrow{AB}} = z_B - z_A$; $z_{\overrightarrow{OC}} = z_C - z_O$

$$\frac{z_C - z_O}{z_B - z_A} = \frac{(1+\sqrt{3}) + i(\sqrt{3}-1)}{(\sqrt{3}-i) - (1+i\sqrt{3})} = \frac{(1+\sqrt{3}) + i(\sqrt{3}-1)}{(\sqrt{3}-1) + i(-\sqrt{3}-1)}$$